Homework 3 Solutions

Problem 1

Look up the specific volume and enthalpy for the inlet conditions.
Specific volume = 0.16849 m3/kg and Enthalpy = 3251.6 kJ/kg
(a) Use the volumetric flow rate, area and specific volume to determine the inlet velocity = **42.1 m/s**

Perform a first law balance over the control volume (note that PE and W are zero)
From the balance determine the enthalpy of the exit stream, which is equal to 3211.9 kJ/kg

(b) From the tables (given the exit pressure and enthalpy) determine the exit temperature = **378.6^{o}C**

Problem 2

Take the turbine as the system (control volume).
At the inlet conditions, and from the two intensive properties specified, the following information can be obtained:
Steam is superheated and the enthalpy at the inlet is equal to 3247.6 kJ/kg

At the turbine exit, it is clear that a saturated liquid stream emerges and at 15 kPa (given the quality), the enthalpy can be calculated to be equal to 2361.01 kJ/kg

From the information given:

h_i = 3283.4 kJ/kg and h_e = 2361.01 kJ/kg, thus Δh = -886.6 kJ/kg
KE_i = 1.25 kJ/kg, KE_e= 16.2 kJ/kg, thus ΔKE = 14.95 kJ/kg
PE_i = 0.0981 kJ/kg, PE_e= 0.0589kJ/kg, thus ΔPE = -0.0392 kJ/kg

(b) w = -[Δh + ΔKE + ΔPE] = 871.7 kJ/kg

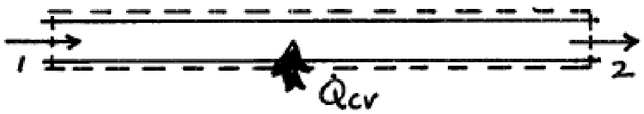
(c) The required mass flow rate for a 5 MW power plant is = 5000 kJ/s/ 871.7 kJ/kg = 5.74 kg/s

Two observations can be made from these results. First, the change in potential energy is insignificant in comparison to the changes in enthalpy and kinetic energy. This is typical for most engineering devices. Second, as a result of low pressure and thus high specific volume, the steam velocity at the turbine exit can be very high. Yet the change in kinetic energy is a small fraction of the change in enthalpy (less than 2 percent in our case) and is therefore often neglected.

Problem 3

SCHEMATIC & GIVEN DATA:

carbon dioxide
$p_1 = 2$ bar
$T_1 = 300$ K
$V_1 = 100$ m/s



$p_2 = 0.9413$ bar
$V_2 = 400$ m/s
$c_p = 0.94$ kJ/kg·K
$d = 2.5$ cm $= 0.025$ m

ASSUMPTIONS: (1) The control volume is at steady state. (2) $\dot{W}_{cv} = 0$. (3) The duct has constant area. (4) Potential energy change from inlet to exit can be neglected. (5) The carbon dioxide can be modeled as an ideal gas with constant specific heats.

To fix the exit state, begin with the steady-state mass balance

$$\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$$

$$\frac{\cancel{A_1} V_1}{v_1} = \frac{\cancel{A_2} V_2}{v_2} \Rightarrow v_2 = \left(\frac{V_2}{V_1}\right) v_1$$

with the ideal gas equation of state

$$\frac{\cancel{R} T_2}{p_2} = \left(\frac{V_2}{V_1}\right)\frac{\cancel{R} T_1}{p_1} \Rightarrow T_2 = \left(\frac{V_2}{V_1}\right)\left(\frac{p_2}{p_1}\right) T_1 = \left(\frac{400}{100}\right)\left(\frac{0.9413}{2}\right)(300\text{K})$$

$$= 564.8\text{ K}$$

Evaluating the mass flow rate

$$\dot{m} = \frac{A V_1}{v_1} = \frac{\left(\frac{\pi d^2}{4}\right) V_1}{(R T_1 / p_1)}$$

$$= \frac{\left(\frac{\pi (.025^2 \text{ m}^2)}{4}\right)(100 \text{ m/s})(2 \text{ bar})}{\left(\frac{8.314}{44.01}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(300\text{K})}\left|\frac{10^5 \text{ N/m}^2}{1 \text{ bar}}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}}\right|$$

$$= 0.1732 \text{ kg/s}$$

Using the steady-state energy balance to find the heat transfer rate

$$0 = \dot{Q}_{cv} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\cancelto{0}{(z_1 - z_2)}\right]$$

with $h_1 - h_2 = c_p (T_1 - T_2)$

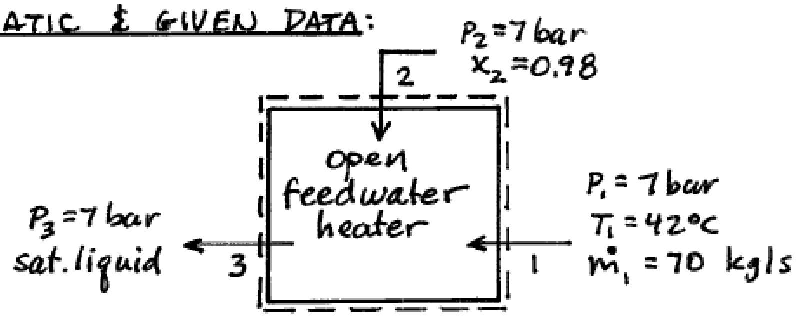
$$\dot{Q}_{cv} = \dot{m}\left[c_p (T_2 - T_1) + \left(\frac{V_2^2 - V_1^2}{2}\right)\right]$$

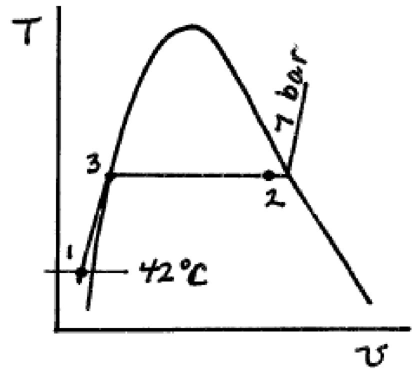
$$= \left(0.1732 \frac{\text{kg}}{\text{s}}\right)\left[\left(0.94 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(564.8 - 300)\text{K} + \left(\frac{400^2 - 100^2}{2}\right)\frac{\text{m}^2}{\text{s}^2}\left|\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}}\right|\right]\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$

$$= 56.1 \text{ kW} \longleftarrow \dot{Q}_{cv}$$

Problem 4

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer with the surroundings is negligible, and $\dot{W}_{cv} = 0$. (3) Kinetic and potential energy effects are negligible.

To find $\dot{m}_2$, begin with the steady-state mass and energy balances

$$0 = \cancel{\dot{Q}_{cv}}^{\,0} - \cancel{\dot{W}_{cv}}^{\,0} + \dot{m}_1\left(h_1 + \frac{V_1^2}{2} + gz_1\right) + \dot{m}_2\left(h_2 + \frac{V_2^2}{2} + gz_2\right) - \dot{m}_3\left(h_3 + \frac{V_3^2}{2} + gz_3\right)$$

With $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$ and assumption (3)

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3$$

or

$$\dot{m}_2 = \dot{m}_1 \left(\frac{h_1 - h_3}{h_3 - h_2}\right)$$

From Tables B it can be ascertained that state 1 is a compressed liquid. Furthermore it can be assumed that:

$$h_1 \approx h_f(T) + v_f(T)\left[p - p_{sat}(T)\right]$$
$$= 175.9 \text{ kJ/kg} + \left(1.0086 \times 10^{-3} \frac{m^3}{kg}\right)\left[7 - 0.08268\right]\text{bar}\left|\frac{10^5 \text{ N/m}^2}{1 \text{ bar}}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}}\right|$$
$$= 176.6 \text{ kJ/kg}$$

And that

$$h_2 = h_{f2} + x_2 h_{fg2} = 697.22 + (.98)(2066.3) = 2722.2 \text{ kJ/kg}$$
$$h_3 = 697.22 \text{ kJ/kg}$$

Finally

$$\dot{m}_2 = (70 \text{ kg/s})\left(\frac{176.6 - 697.22}{697.22 - 2722.2}\right)$$
$$= 18.0 \text{ kg/s}$$

$\dot{m}_2$

Problem 5

For the **co-current heat exchanger**, the heat duty is given by:
Flow rate of hot stream * Specific Heat Capacity of hot stream * temperature difference for hot stream=
(7/2)*8.314*(1/1000)*(400-350) = **1.45 kW**

This duty is also equal to
Flow rate of cold stream * Specific Heat Capacity of cold stream * temperature difference for cold stream =
$\dot{m}$*(7/2)*8.314 * (340 – 300)

Therefore the flow rate of the cold stream in the co-current heat exchanger will be equal to: **1.25 mol/s**

For the **counter-current heat exchanger**, the heat duty identical to that calculated for the co-current heat exchanger = **1.45 kW**

Using the same argument as above, the flow rate of the cold stream in the counter current heat exchanger
$\dot{m}$*(7/2) * 8.314*(390 – 300) = 1,450
Therefore, the flow rate of the cold stream = **0.554 mol/s**

The counter current configuration is more efficient than the co-current heat exchanger (for the same cooling heat duty, a lesser amount of cooling fluid is required)